Why d^2-law does not hold during droplet vaporization in microgravity conditions?

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Abstract
The d^2-law validity during droplet vaporization in microgravity environment is examined experimentally. The same experiment is performed in normal and microgravity environments. The droplet, which is exposed to hot stagnant ambient, is suspended onto the cross point of two micro fibers of 14 micrometers in diameter. The ambient temperature is varied in the range up to 967 K. The results show that, for ambient temperatures below approximately 950K, the d^2-law departs during droplet vaporization in microgravity environment, whereas it is maintained in normal gravity. In addition, the droplet lifetime is longer in microgravity than in normal gravity under the same test conditions. However, for temperatures exceeding approximately 950K, the experimental results demonstrate that the d^2-law holds throughout the entire droplet lifetime, and the droplet lifetime is identical in both environments.

Introduction

Droplets gasification is an important process for the performance of liquid-fuelled combustion systems, such as diesel and propulsion engines. In these systems, the fuel is sprayed as a cloud of droplets in the combustion chamber of, for example, diesel or gas turbine engines, which then vaporize and oxidize to release heat. Therefore, the study of the gasification of a droplet via vaporization, which involves the heat, mass and momentum transfer processes in gas and liquid phases, and their coupling at the droplet interface, would provide the knowledge base for better understanding and modeling complex spray flows. The pioneering work of Godsave in the early 1950s [1] has led to the development of a theoretical model that is capable of describing the gasification process of a droplet. This model, which is termed "Quasi-Steady model" and also called d^2-law, demonstrates that during the gasification process, the droplet surface area, represented by the droplet-squared diameter, varies linearly during its lifetime. Although this model is very successful in describing the gasification process of a fuel droplet, the assumptions upon which the model has been developed are subjected to several experimental and numerical analyses (see, for example, thorough review articles [2-6]). Amongst the most controversial assumptions, which are reviewed in several publications (see, for example, [3-4], and references cited therein), are evoked below due to their pertinence to the present study.

(a) Constant and uniform droplet temperature. The assumption of constant and uniform droplet's temperature has quickly been subjected to several studies (see Refs. [7-13] to cite only a few). These studies and others demonstrate the existence of a transient heat-up period prior to the occurrence of the d^2-law. During the initial stage of the droplet lifetime, i.e. the non-steady period, it is discovered that the heat received initially by the droplet is utilized to heat-up the liquid to its equilibrium temperature, i.e. wet bulb temperature. Once this temperature is achieved, then all the heat transferred into the droplet is employed to gasify the liquid. However, the need for experimental data to verify the numerical predictions of the existence and lifetime of the droplet heat-up period complicated the situation, as the fiber (or the droplet suspending material) would induce extra effect, which is not part of the physical problem, and hence alter the droplet heat transfer. In an experimental study of the vaporization process, in general, the droplet is suspended on the tip of a thin fiber. Several experimental and numerical studies have been performed to assess the influence of the fiber, and consequently the corresponding heat transfer (see, for example, Refs. [14-16] and references cited therein). It is proved that the fiber can indeed influence the droplet heat and mass transfer. Numerical studies of the gasification of a droplet in the absence of a suspending fiber confirm the existence of a non-steady heat-up period prior to the occurrence of the gasification process (see for example a recent publication [17]). In order to completely eliminate the fiber effect on droplet gasification, a new experimental approach has recently been recently developed [18-19]. It consists of two micro-sized fibers where the droplet is suspended at the cross point. The results, which are presented below, confirm that the d^2-law does not hold during this period of the droplet lifetime.

(b) Spherical symmetry means that convection (either natural or forced) is absent and hence droplet remains spherical during its lifetime. This assumption made it easy to develop the d^2-law. However, under certain practical applications, the droplet deforms during its gasification due to the presence of natural convection or a strong forced flow and thus the spherical symmetry assumption breaks down. However, numerous experimental and numerical studies are undertaken to verify the implication of this assumption. The results are

that the d^2-law still holds even under convection conditions but only once the droplet reaches its thermal equilibrium. It must be recalled that the idea behind this assumption is to avoid a complicated/lengthy numerical solution and rather develop a simple analytical solution based on spherical symmetry and then correct for the effect of convection by using empirical correlations [6]. In addition, the assumption of spherical symmetry of the droplet is important for ensuring a spherical envelop flame.

(c) The gas-phase quasi-steadiness, which indicates that the gas-phase immediately adjusts itself to the local boundary conditions and droplet size at each instant of time. Although this assumption seems unrealistic in the early years of d^2-law, it is only relatively recently when it is discovered the deficiency of the d^2-law as a result of non-steadiness of the gas-phase surrounding the droplet. (See, for example, [20-22]). It is pointed out that the effects of this assumption on the d^2-law are not yet completely identified [3].

The principal objective of the present study is to further examine the d^2-law under well-controlled test conditions in normal and micro-gravity environments.

Experimental Set-up and Test Conditions

The experimental set-up is reported elsewhere [19]. Therefore, only a brief reminder is provided here. The experimental test facility consists mainly of a furnace and a droplet's support and formation system. The furnace is essentially a short cylinder of an inner diameter of 68 mm and 100 mm height, which results in a volume of 360 cm^3. The furnace is capable of attaining a temperature of up to 1200 K, which is generated by Joule effect placed in a pressure chamber which surrounds the furnace. The droplet, which is supported on a "cross-fiber" system, is formed by injecting n-decane liquid, via a piezo-electric injection system, at the intersection point of the two quartz fibers, which are fixed perpendicularly by a metallic frame system. The quartz fibers/wires are 14 μm in diameter. Decane ($C_{10}H_{22}$) is chosen as the liquid because it is relatively non-volatile fuel and consequently allows sufficient time for setting up and performing the experiment. The furnace gaseous medium is chosen to be Nitrogen to avoid oxidation. The droplet is initially formed in the lower part of the chamber, and then introduced into the furnace by the aid of a motorized displacement system. Once the droplet is exposed to the hot atmosphere in the furnace, the temporal regression of the droplet surface is recorded by using a high-speed video camera with a frame rate that can be varied between 150 and 750 fps. For each set experiment, approximately 700 images are recorded in order to allow for a satisfactory temporal resolution. In addition, at least three experiments are performed for each test conditions to verify the repeatability of the data as well as to remove statistical errors. The images are transferred to a computer which are then analyzed in the post-processing to deduce the droplet instantaneous surface area and hence its diameter temporal variation.

Note that the calculation error in determining the droplet diameter is of the order of ± 3%. As an example, the droplet vaporization rate for T_∞ = 767K lies between 0.183 mm^2/s and 0.190 mm^2/s, which results in uncertainty of ± 2%. Two series of droplet vaporization experiments are carried out in the present study; one in normal gravity, and another in microgravity conditions of $10^{-2}g_0$. The latter are realized via parabolic flights aboard of the A300 Aircraft of CNES. Note that the parabolic flight duration is of the order of 22 seconds. This time duration is relatively short which prevent experiments at ambient temperatures below approximately T_∞ ~ 500 K. In both series of experiments, the pressure in the furnace is kept atmospheric while the temperature is varied in the range between 567K and 967K for microgravity experiments, and between 473K and 967K for normal gravity experiments. The droplet diameter is kept equal approximately 370 μm for the microgravity experiments, and around 500 μm for normal gravity experiments.

Results - Effect of the Droplet Suspending Technique

The influence of the fiber's size on the d^2-law during droplet evaporation is investigated by conducting experiments at a typical ambient temperature, T_∞ = 570 K, in normal gravity environment. Two different droplet's supporting techniques are employed, which are the cross-fiber technique and the classical suspending fiber technique. For the cross-fiber technique, the fiber's diameter is 14 μm, whereas for the suspending technique three different diameters of the fiber are tested, which are d_f = 106 μm, d_f = 144 μm and d_f = 181 μm. The initial droplet diameter is kept constant, d_0 = 800 μm, in all experiments to eliminate the effect of the droplet diameter (i.e. buoyancy effect). Figure 1a shows, as expected, that the droplet is ellipsoid in shape when suspended on a classical fiber, whereas it is nearly spherical when suspended on the cross micro-fibers, as shown in Figure 1b.


(a)

(b)
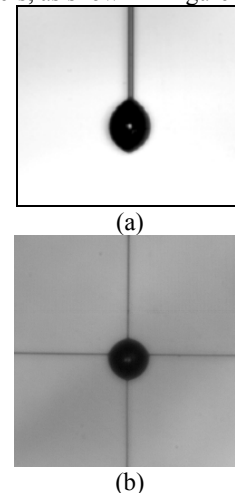
Fig.1. Droplet's suspending (a) classical fiber, and (b) cross-fiber technique.

The time history of *n*-decane droplet is plotted in Figure 2 for the three single fibers as well as for the cross fiber technique. This figure reveals three distinct observations. Firstly, the droplet heat-up period occurs even with the cross-fiber technique though it is shorter. Secondly, for the same droplet diameter ($d_0 \sim 800$ μm), the droplet lifetime shortens as the fiber diameter increases, which is an indication of the enhancement of the droplet heat and mass transfer rates (note that the droplet vaporization rate is calculated as $K = -d(d^2)/dt$). Thirdly, the most remarkable observation, concerning the effect of the classical fiber technique, is the enhanced vaporization rate during the final part of the droplet lifetime during which the fiber's influence on droplet heat and mass transfer intensifies, and hence the d²-law breaks down during the last portion of the droplet lifetime. Contrary to the case of the classical fiber, in the case of the cross-fiber technique the droplet vaporization continues until the complete depletion of the droplet (i.e. when $d \sim 0$). However, the temporal d²-variation becomes not linear towards the end of droplet lifetime during which the droplet evaporation process appears to slow down relatively. It is important to note that this phenomenon appears only when suspending large droplets on the cross-fiber system; however, the d²-law is maintained throughout the droplet lifetime with relatively smaller droplets, as can be seen in Figure 3. This figure shows also that the lifetime of the heat-up period almost disappears for small droplets. Further investigations are underway to explore the link between the droplet size and d²-law in the case of the cross-fiber suspending technique.
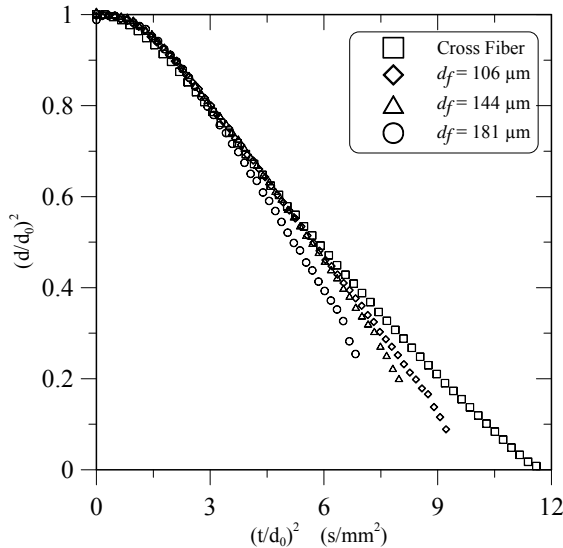
Fig. 2. *n*-decane droplet time history ($p_\infty = 0.1$ MPa, $T_\infty = 570$ K, $d_0 = 800$ μm)
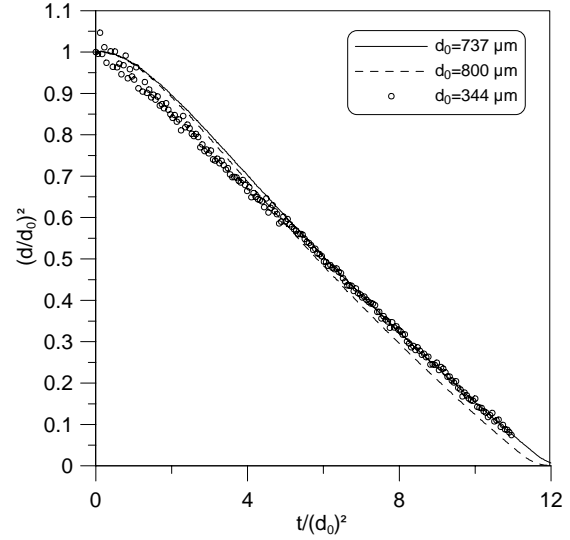
**Fig. 3**. *n*-decane droplet time history for various initial diameters ($p_\infty = 0.1$ MPa, $T_\infty = 570$ K)
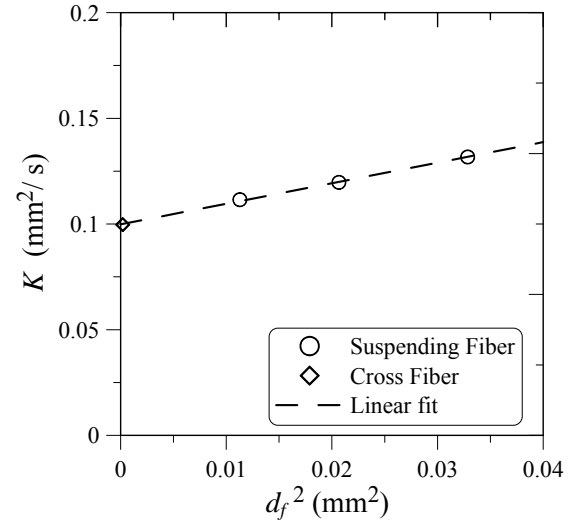
Fig. 4. *n*-decane droplet vaporization rate versus the suspending fiber size ($p_\infty = 0.1$ MPa, $T_\infty = 570$ K, $d_0 = 800$ μm).

The corresponding droplet average-vaporization rates are plotted in Figure 4 as a function of the squared diameter of the quartz fiber, d_f^2. This figure shows that the droplet vaporization rate increases linearly with the fiber cross-sectional area (represented by the fiber squared diameter). This implies that the conduction heat flux into the droplet is proportional to the fiber cross-sectional area, and subsequently to d_f^2. Therefore, the extrapolation of the linear relationship between the fiber cross-section and the droplet vaporization rate suggests that the value of K_0 corresponding to $d_f^2 = 0$ is free from the fiber effect. This value is found practically identical to that obtained by using the cross-fiber technique. This suggests that the droplet supporting cross-fiber technique, which is employed in the present

experiments, does not cause any interference with the droplet heat transfer, and hence its evaporation process.

Results - Effect of Natural Convection
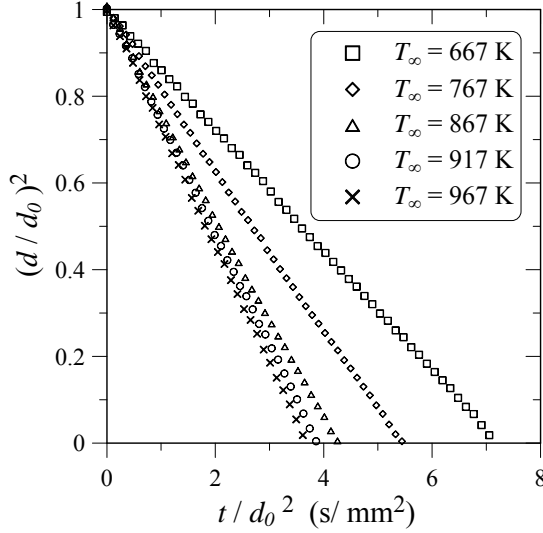
Fig. 5. Time history of *n*-decane droplet in normal gravity environment (p_∞ = 0.1 MPa, d_0 = 500 μm).
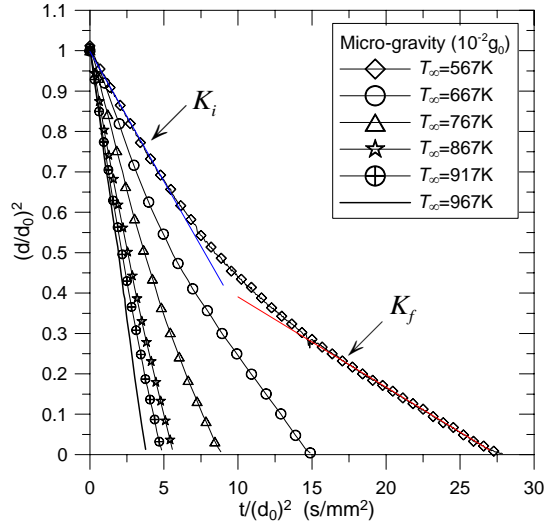
Fig. 6. Time history of *n*-decane droplet in microgravity environment (p_∞ = 0.1 MPa, d_0 = 370 μm).

Figures 5 and 6 present the time-history of the normalized squared-diameter of *n*-decane droplet for various different ambient gas temperatures under normal gravity and microgravity conditions, respectively. Note that these figures do not exhibit the droplet heat-up period. During this period the droplet is moved from the chamber into the furnace (See Figure 1). It is observed that this maneuvering is in fact not free of vibrations and consequently induce oscillations [23]. To cut-off these oscillations, it is decided to start recording the surface area of the evaporating droplet once the droplet is stabilized after which the heating period is already elapsed particularly at very high ambient temperatures in which the heat-up period is almost inexistent for relatively small droplets ($d_0 \sim <$ 400 μm). Figure 6 reveals that as the droplet is exposed to a sufficiently hot gaseous environment, it starts evaporating relatively faster during the initial period of its lifetime, for which the d^2-law holds, which is then followed by a gradual slow down in the droplet evaporation rate during which the d^2-law appears to hold again. In between, the two linear temporal variations of the droplet squared diameter, which can be characterized by the d^2-law, there exists a non-steady transitional region in which the d^2-law breaks down. However, as the ambient temperature increases the "non-linearity" or "departure" of the d^2-law diminishes until the d^2 regains its full linearity variation during the entire droplet lifetime. On the other hand, in normal gravity environment, the d^2-law holds for the same test conditions as in microgravity. Moreover, at ambient temperatures, T_∞ below approximately 950 K, the constant of the slop of the d^2 temporal variation during the initial part of the droplet lifetime (referred to as K_i), which is the highest in microgravity conditions, is lower than its counterpart in normal gravity conditions. It can also be observed that the value of K_i increases with T_∞ until it becomes identical to its counterpart value in normal gravity conditions once the droplet vaporization process obeys completely the d^2-law in microgravity conditions (compare the d^2-variation of Figure 5 with its counterpart of Figure 6 at $T_\infty \sim$ 967K).

An attempt is provided below to explain the differences in the time evolution of d^2 between normal and micro- gravity conditions, as well as to discuss the failure of d^2-law in microgravity environment. This discussion is based on the concept of the vapor's residence characteristic time around the droplet surface. In fact, there are two characteristic times; one for the flow field engendered by the natural convection and the other for the flow field caused by the radial velocity, which may be defined, respectively, as $t_{NC} = r_s / u_{NC}$ and $t_{u_r} = r_s / u_r$ where r_s is the instantaneous droplet radius, and u_{NC} and u_r are the natural and radial velocities, respectively. Their definitions are reported in [23-26]

At ambient temperatures, T_∞, below ~ 950 K, the fact that the droplet's evaporation is slower in microgravity conditions in comparison with its counterpart in normal gravity may be attributed to the higher flow velocity responsible for the vapor evacuation away from the droplet surface. In normal gravity, the flow radial velocity is negligible compared to its counterpart due to natural convection [23]. In microgravity conditions of $10^{-2}g_0$, on the other hand, the radial velocity is larger than its counterpart's velocity induced by natural convection at around $T_\infty > 950$, whereas these two velocities have similar magnitude at the lower end of the ambient temperature range (i.e. $T_\infty \sim$ < 950 K) [23]. The obtained corresponding characteristic times of the vapor in the vicinity of the droplet surface show that the vapor residence time is

much smaller in normal gravity environment for any given ambient temperature within the explored range. This suggests that the vapor residence time in micro-gravity conditions is relatively long which allows for the vapor build-up on the droplet surface. The vapor accumulation in the droplet vicinity induces a sort of resistance against the heat transfer to the liquid phase, and consequently reduces the droplet's surface temperature. This argument seems in line with that of Wu et al. [20], Krier and Wronkiewicz [21], and Waldman [22] who investigate droplet combustion and report that the droplet gasification intensifies as the flame approaches the droplet surface, and slows down as the flame moves away from the droplet surface. They attribute this phenomenon to the vapor accumulation on the droplet surface. An analogy between a droplet surrounded by a spherical flame and a hot atmosphere encircling a droplet might be used, although in the former there is vapor consummation while it is absent in the latter. Thus, if the droplet ambient is sufficiently hot and the vapor residence time around the droplet surface (or the vapor evacuation characteristic time) is relatively long then the vapor build-up would occur, which in turn reduces the heat being received by the liquid phase, which, in turn, results in a lower droplet mass transfer rate. It is important to note that during this initial part of the droplet lifetime, the d^2-law is preserved. However, the continuing process of the vapor build-up leads to an increased loss of the droplet heat through the vapor layer that is built around the droplet surface, which then results in a non-steady process of the gas phase. As soon as a thermal equilibrium is established between the droplet and its surrounding gaseous medium, the radial velocity becomes capable of evacuating the mass of the vapor being formed away from the droplet surface. This is clearly indicated by a decrease in the vapor residence time [23]. During this stage of the process, the droplet surface temperature is believed to remain constant and hence the d^2-law appears to hold for the rest of the remaining droplet lifetime.

At ambient temperatures, T_∞, beyond ~ 950 K, the d^2-law holds in normal as well as in micro-gravity conditions, and consequently the droplet lifetime becomes identical in both cases. It appears that at these ambient temperatures, the radial velocity, which is a characteristic of the vapor evacuation velocity, attains a critical value beyond which the vapor build-up on the droplet surface is prevented or minimized by convecting the formed vapor away from the droplet surface. Note that at this critical velocity the vapor residence time in microgravity conditions becomes relatively comparable in order of magnitude with that in normal gravity conditions (see Table 1). Therefore, the heat flux into the liquid phase is kept constant which thus maintains the droplet "wet bulb" temperature, and hence a constant mass transfer rate in both environments, i.e. normal and reduced gravity, respectively. Accordingly, it might be possible to postulate the following: (i) in the absence of any forced convection, the role of natural convection is to induce a flow velocity responsible for

evacuating the formed vapor away from the droplet surface, and hence preventing vapor accumulation; (ii) in the absence of natural convection (or weak convection in general), there would be always vapor accumulation as long as the flow radial velocity is below a so-called "flow critical velocity" which is needed to prevent significant vapor build-up. This suggests that once this flow's critical velocity is attained; the effect of natural convection becomes unimportant as the corresponding flow's radial velocity becomes significantly influential. Contrary to the process of droplet vaporization, in the case of droplet burning, the radial velocity is much larger than the aforementioned "flow's critical velocity" due to enhanced molecular diffusion. This is why the d^2-law holds for droplet vaporization in microgravity at ambient temperatures beyond a certain value, and fails otherwise.

T_∞ (K)	Normal gravity ($1g_0$)	Microgravity ($10^{-2}g_0$)
667	~ 1.5 (ms)	~ 16.4 (ms)
967	~ 1.3 (ms)	~ 4.0 (ms)

Table 1. Vapor's residence characteristic time at the onset of the droplet evaporation process for typical ambient temperatures in normal and microgravity environments

Conclusions
The present study examines the d^2-law under well-controlled normal and micro-gravity conditions. The effect of gravity is studied by performing experiments in normal gravity and microgravity environments while keeping all other test conditions identical; that is, constant atmospheric pressure, and ambient/furnace temperature in the range up to around 970K. The effect of the droplet suspending technique is eliminated by using a novel cross micro-fiber system. This technique enables to preserve the spherical shape of the droplet throughout the vaporization process in normal gravity atmosphere. In the absence of any forced convection, the experimental results show that the d^2-law always holds in normal gravity within the range of the ambient temperature explored in the present study. However, under similar ambient test conditions, the d^2-law fails in microgravity environment at ambient temperatures below ~ 950 K. An explanation of the failure of the d^2-law is provided in terms of a "characteristic velocity" of the flow field around the surface of the droplet, which is responsible for evacuating the vapor away from the droplet surface. This also can be translated in terms of the corresponding "characteristic time" of the vapor residence in the vicinity of the droplet surface. This approach suggests that there exists a "critical value" of the flow field velocity, which is responsible for vapor evacuation from the droplet surface, beyond which a significant vapor build-up around the droplet may be avoided, and hence the d^2-law can hold.

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