Droplet Transport Rates in a Turbulent Hot Airflow

Maher M. Abou Al-Sood[a], Madjid Birouk[a*] and Iskender Gökalp[b]
[a]Department of Mechanical and Manufacturing Engineering
University of Manitoba, Manitoba, Canada
[b]Institut de Combustion, Aerothermique, Reactivite et Environment (ICARE)
CNRS, Orleans, France

Abstract
A three-dimensional numerical model is developed to assess the effect of turbulence on heat and mass transfer of a single droplet exposed to a freestream of air. The freestream temperature, turbulence intensity and Reynolds number are varied to provide a wide range of test conditions under standard ambient pressure. To account for high temperature conditions, variable thermophysical properties, gas and liquid phase transients and radiation are considered. The turbulence terms in the conservation equations of the gas-phase are modeled by using the shear-stress transport (SST) model. A Cartesian grid based blocked-off technique is used in conjunction with the finite-volume method to solve numerically the governing equations of the gas and liquid-phases. The numerical results indicate that the effect of freestream turbulence is persistent although it weakens as the airstream temperature increases. The effect of radiation becomes significantly important at elevated airstream temperatures. Comprehensive droplet heat and mass transfer correlations are proposed, which take into consideration all the aforementioned variables.

Introduction

Droplet evaporation process is one of the controlling processes for the design, performance and emission of liquid-fuelled combustion systems. Describing the process of droplet transport rates by using simple correlations is important for modeling spray combustion. The most recognized published droplet heat and mass transfer correlations are laminar. This is may be due to the lack of reliable turbulent droplet heat and mass transfer data. Indeed, a recent review of the literature of the early and recent studies on the effect of turbulence on droplet/sphere heat and mass transfer reveal the inconsistency of the published data [1]. The major conclusion of the review is that the majority of the early studies claimed an increase in sphere/droplet heat and mass transfer rates due to turbulence, although a consistent trend is lacking. Whereas other studies claim that turbulence has negligible effect on droplet/sphere heat and mass transfer. The review by Birouk and Gökalp [1] reports that most of the studies, which claim that turbulence enhances droplet/sphere heat and mass transfer, correlate the effect of turbulence on sphere/droplet heat and mass transfer in terms of Nusselt (Nu) and Sherwood (Sh) numbers, respectively. The general form (Eqs. (1)-(2)) of these expressions are adopted from the laminar correlations with the inclusion of a new term, C_T, to represent turbulence [1-3].

$$Sh = A' + B'\,\mathrm{Re}^{1/2}\,Sc^{1/3}\left(C_T\right) \quad (1)$$

$$Nu = A' + B'\,\mathrm{Re}^{1/2}\,\mathrm{Pr}^{1/3}\left(C_T\right) \quad (2)$$

where A' and B' are constants. However, there is a significant disparity of the values of the aforementioned coefficients (A', B', C_T) between the different authors. It is concluded in [1] that the level of turbulence employed by different researchers is, in fact, very low and may be within their experimental uncertainties.

The review by Birouk and Gökalp [1] show also that in recent studies, attempts have been made to correlate the effect of turbulence on droplet mass transfer in terms of an effective vaporization Damköhler number [4-6]. Recently, Abou Al-Sood [2] and Abou Al-Sood and Birouk [3] developed a numerical study to take part of the aforementioned debate by employing a wide range of test conditions. The results are that the Damköhler number correlation is found not applicable in its current form for correlating the droplet turbulent evaporation at high-temperature atmospheric conditions [3]. Therefore, Eq. (1) has been by Abou Al-Sood and Birouk [3] to develop a correlation which accounts for the effect of freestream turbulence on droplet mass transfer. This correlation, which is developed over a wide range of test conditions, is expressed as follows:

$$Sh_f\,(1+B_{M,f})^{0.7} = 2 + 0.914\,\mathrm{Re}_M^{1/2}\,Sc_f^{1/3}\left(1+1.235 I_\infty^{0.372}\right) \quad (3)$$

where B_M, Re, Sc and I are the mass transfer number, Reynolds number, Schmidt number and turbulence intensity, respectively. The subscripts f and ∞ denote film and free stream conditions. Although this correlation worked well for a wide range of turbulent flow and liquid fuel properties, the effects of radiation and ambient pressure, which can be important, are neglected. Therefore, the aim of the present study is extend the applicability of this mass transfer correlation by taking into consideration the effect of radiation. In addition, an attempt is made to develop a droplet heat transfer correlation in terms of Nusselt number based on Eq.(2).

[*] Corresponding author: biroukm@cc.umanitoba.ca
Associated Web site: http://umanitoba.ca/faculties/engineering/
Proceedings of the 21th ILASS – Europe Meeting 2007

Mathematical model

Description of the physical model and assumptions

The physical problem consists of a liquid droplet, with an initial radius of r_0 and an initial uniform temperature T_0, which is immersed in a turbulent inert airstream of infinite expanse. The gas-phase is prescribed by its freestream mean velocity, U_∞, pressure, p_∞, temperature, T_∞, fuel mass fraction, $Y_{F\infty}$, turbulence intensity, I_∞, turbulence kinetic energy, k_∞ and its dissipation rate per unit of turbulence kinetic energy, ω_∞.

The following assumptions are employed in the present model: (*i*) the droplet is stationary and consists of a single chemical component, (*ii*) the droplet shape remains spherical because the droplet Weber number is much less than unity, (*iii*) the droplet evaporates in an inert atmosphere, (*iv*) the gas-liquid interface is at an equilibrium phase, (*v*) gravity, Dufour (energy flux due to mass concentration) and Soret effects (mass diffusion due to temperature) are assumed negligible, and (*iv*) radiation is considered with the assumption that the gas-phase between the droplet and the wall is transparent and does not emit any radiation, and the wall, which is chosen here as the boundary of the calculation domain, is assumed as blackbody emitter with a temperature equals to that of the freestream.

Governing equations

The governing equations for the gas-phase are the Reynolds-Averaged Navier-Stokes (RANS), energy and mass species conservation equations. Details of these equations with the adopted turbulence closure model, i.e. shear stress transport (SST) model of Menter [7], are reported in previous publications [2, 3, 8]. For the liquid phase (i.e. droplet), the governing equations are basically the unsteady continuity, momentum and energy equations.

Radiation heat transfer model

Since the droplet is large enough, an approximation of the radiation absorption that occurs only at the droplet surface with an effective surface absorptance α_{eff} (as $\dot{q}_r = \alpha_{eff}\sigma(T_\infty^4 - T_s^4)$) is considered in this study. The value of the effective surface absorptance is based on the data of Tseng and Viskanta [9], which is a function of the droplet diameter and ambient temperature. In the present study, the value of α_{eff} for *n*-decane, which is not readily available in the open literature, is assumed approximately equal to that of diesel fuel.

Freestream and gas-liquid interface conditions

The freestream mean velocity components, pressure, temperature, fuel mass fraction and turbulence quantities at the inlet of the computational domain are taken as $u = U_\infty, v = 0$, $w = 0$, $p = p_\infty$, $T = T_\infty$, $Y_F = 0$, $k = k_\infty$ and $\omega = \omega_\infty$. The freestream k_∞ and ω_∞ are estimated by using the following relations as $k_\infty = 1.5\left(I_\infty \times U_\infty\right)^2$ and $\omega_\infty = \rho_\infty\left(k_\infty/\mu_\infty\right)\left(\mu_{t\infty}/\mu_\infty\right)^{-1}$ where $\mu_{t\infty}$ is the freestream turbulent viscosity which is taken as $\mu_{t\infty} \cong \mu_\infty$ [7, 10]. A distinctive gas-liquid interface exists at the droplet surface and conditions at this interface are obtained by coupling the conservation equations (momentum, energy and species equations) in the gas and the liquid-phases as follows [2-3]

a) Shear stress continuity

$$\tau_{ij,g} = \tau_{ij,l} \tag{4}$$

b) Tangential velocity continuity

$$U_{\tan}\big|_g = U_{\tan}\big|_l = U_s \tag{5}$$

c) Normal velocity continuity

$$U_{nor}\big|_l = \left(\frac{\rho_g}{\rho_l}\right)U_{nor}\big|_g + \dot{r}\left(1 - \frac{\rho_g}{\rho_l}\right) \tag{6}$$

d) Temperature continuity

$$T_g = T_l = T_s \tag{7}$$

e) Energy conservation

$$\lambda\frac{\partial T}{\partial x_i}\bigg|_l = \lambda_{eff}\frac{\partial T}{\partial x_i}\bigg|_g - \dot{m}''_{evap}h_{evap} + \dot{q}_r \tag{8}$$

f) Species conservation

$$\dot{m}''_{evap}\left(Y_{F,g} - 1\right) - \rho_g D_{AB,g}\frac{\partial Y_{F,g}}{\partial x_i} = 0 \tag{9}$$

g) Droplet mass conservation

$$\dot{r} = -\frac{\sum\limits_{\substack{evaporated\\surfaces}}\dot{m}''_{evap,i}A_s}{4\pi\rho r^2} + \frac{r}{3\rho}\frac{d\rho}{dt} \tag{10}$$

where the subscripts *g* and *l* denote any variable in the gas and liquid sides at the droplet-gas interface, respectively. The parameter $\dot{r}$ denotes the regression rate of the droplet radius, *r* is the instantaneous droplet radius, and A_s is the surface area of the nodes subjected to the flow.

Numerical approach

The finite-volume approach [11] is employed. The governing differential equations are integrated over discrete volumes resulting in a set of algebraic equations having the following general form

$$a_P\Phi_P = \sum a_{nb}\Phi_{nb} + b_\Phi \tag{11}$$

where a_P, a_{nb} and b_Φ are coefficients and their expressions are reported in [2, 8]. The absence of an explicit equation for pressure is dealt with by using the SIMPLE approach [12] in which an expression in the form of Eq. (11) is derived for the pressure through a combination of the continuity and momentum equations. Details about numerical techniques, computational domain, grid generation and independency can be found in [2].

Results

The test conditions include two droplet's diameters: 1.961 mm and 1.5 mm; three freestream mean velocities of 0.6 m/s, 1 m/s and 2 m/s; turbulence intensity in the range between 0% - 60%, and ambient temperature in the range 300K - 1273K. The ambient pressure is kept atmospheric. Formulas which are employed to determine the thermodynamic properties of *n*-heptane and *n*-decane droplets, as well as the vapor-air mixture at the droplet surface vicinity are reported elsewhere [2-3].

Validation of the numerical model

Published data for droplet evaporation in turbulent flow at elevated ambient temperatures are not available. Thus, validation of the present numerical model is performed by comparing the present predictions with their counterparts published numerical and experimental published data for laminar flow conditions. Fig. 1 presents the time-history of the squared normalized diameter of *n*-decane droplet as predicted by the present numerical model. The same figure shows a comparison of the present predictions against the laminar numerical data Megaridis [13], and the laminar experimental data of Wong and Lin [14]. A fair agreement is obtained between the present predictions and the experimental data Wong and Lin [14].

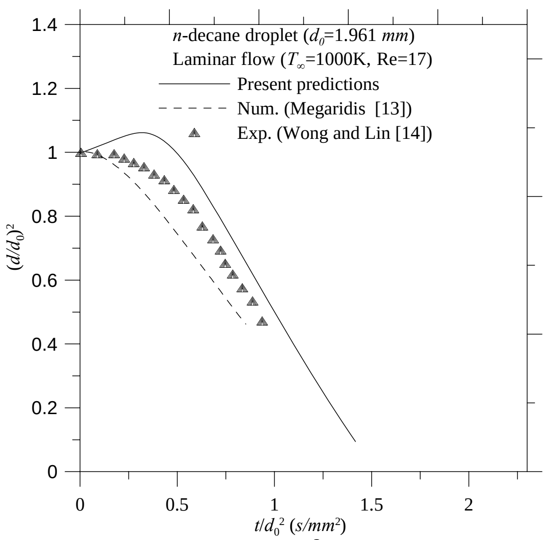

Fig. 1 Time-history of $(d/d_0)^2$ of *n*-decane droplet as predicted by the present model and its comparison with published experimental and numerical data.

Turbulence effect on droplet heat and mass transfer

Fig.2 shows the time history of the normalized squared diameter of *n*-heptane droplet for a freestream mean-velocity of 1 m/s, a wide range of freestream turbulence intensity, and a freestream temperature of 1273 K. Two distinct remarks can be drawn from this figure. Firstly, the heating period becomes shorter as the freestream turbulence intensity increases. Secondly, the total droplet lifetime decreases with increasing turbulence intensity. Furthermore, it appears that the freestream turbulence still has an effect on the droplet's evaporation rate even at high freestream temperature. In addition, Fig. 2 reveals that the d^2-law holds for most of second stage of droplet evaporation (i.e. steady-state evaporation) with the exception of the very end life of the droplet where about 70% of the liquid (i.e. in the region $d<0.3d_0$) is evaporated. This observation agrees with Sazhin *et al*. [15] who claimed that the d^2-law does not hold at higher ambient temperatures (i.e. for $T_\infty >$ 700K) when considering radiation effect.

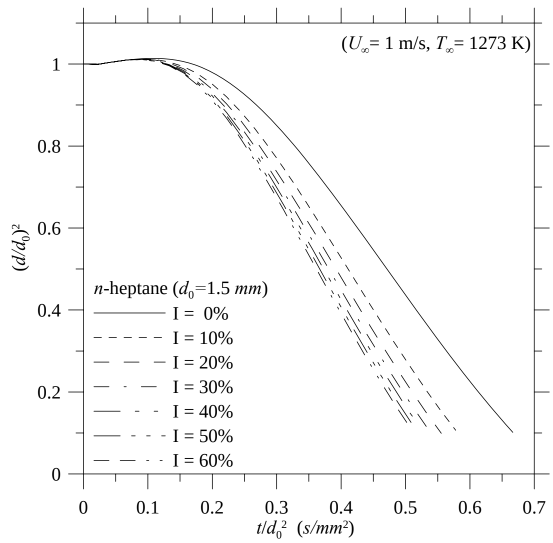

Fig. 2 Time-history of $(d/d_0)^2$ of *n*-heptane droplet at T_∞ = 1273 K and U_∞= 1 m/s for various freestream turbulence intensities

Fig. 3 shows the time history of the surface temperatures of *n*-heptane and *n*-decane droplets at different turbulence intensities. For each droplet the surface temperatures increases until it reaches a maximum value, which is higher than the wet bulb temperatures, after which it decreases again towards the wet bulb temperatures. This trend of the droplet surface temperature, which is related to the effect of thermal radiation, agrees with the predictions of Sazhin *et al*. [15]. At the beginning of evaporation process, the heat transferred to the droplet by convection and radiation is higher than the energy consumed for evaporation which results in an increase in the droplet surface temperature. Another interesting observation in Figure 3 is that the effect of turbulence is noticeable in decreasing the maximum droplet surface temperature. This might be due to the increase of the droplet heat transfer from the gas phase via convection, which, in turn, leads to an increased droplet evaporation. This may result in a decrease in the radiation due to reduced droplet diameter, as $q_r'' \propto d^{2.5}$ [16].

Turbulent *n*-heptane and *n*-decane droplets evaporation rates normalized by their corresponding laminar values, K/K_L, are presented in Fig. 4 for three typical ambient temperatures, i.e. 300K, 773, and 1273K. This figure shows that the effect of turbulence decreases as the freestream temperature increases. For example, the % decrease in K/K_L for *n*-decane due to turbulence is 115.4%, 87.5%, and 79.4% at 300K, 773K and 1273K, respectively. In addition, Fig. 4 shows that the effect of radiation is more pronounced for *n*-decane than *n*-heptane droplet. The same figure shows also that the effect of radiation weakens in the presence of turbulence; for instance, the % decrease in K/K_L of *n*-heptane at 1273K is 27% and 23.1% for a turbulence intensity of 0% (i.e. laminar freestream) and 60%, respectively. Finally, Fig. 4 reveals that the effect of radiation may be neglected at ambient temperatures less than around 800K [17].

Fig. 5 presents a comparison of the turbulent mass transfer correlation which is proposed previously [2-3] with the present predictions, which are obtained under similar test conditions but with the effect of radiation being considered. This figure shows that there is a slight deviation of less than 5%, which occurs especially at relatively lower Reynolds numbers (i.e. at relatively high temperatures; $T_\infty > 800$K).

As the droplet mass transfer is very much coupled with droplet heat transfer, it is also important to develop a correlation for droplet heat transfer, which both are essential for modeling spray combustion. Thus, the data presented above are employed to develop a correlation for heat transfer of a droplet evaporating in a turbulent hot airstream. The test conditions, which are employed here, are the same as for the droplet mass transfer correlation presented in Fig. 5.

The conventional form of droplet heat transfer correlation (Eq. (2)), is adopted. Since the droplet surface temperature does not approach a constant value at higher freestream temperatures due to radiation, the Nusselt number of the droplet steady-state evaporation phase is calculated as the average value of the temporal Nusselt number which corresponds to the steady-state droplet evaporation where the d^2-law holds. The best fit for the present predictions is found to have the following expression (with a standard deviation of 99.5%).

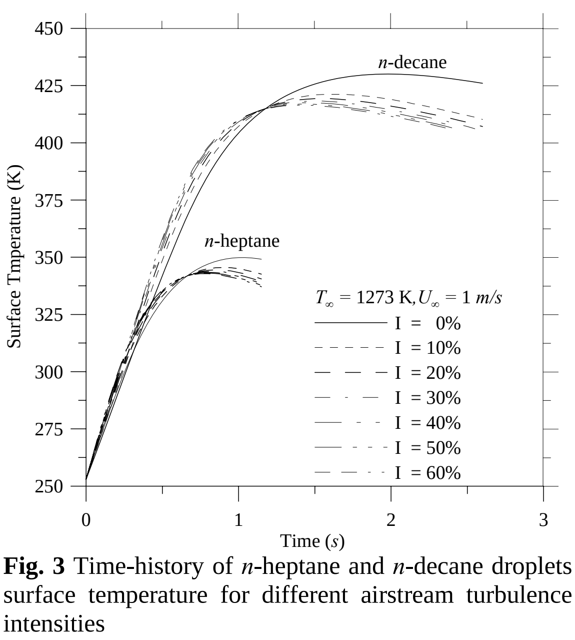

Fig. 3 Time-history of *n*-heptane and *n*-decane droplets surface temperature for different airstream turbulence intensities

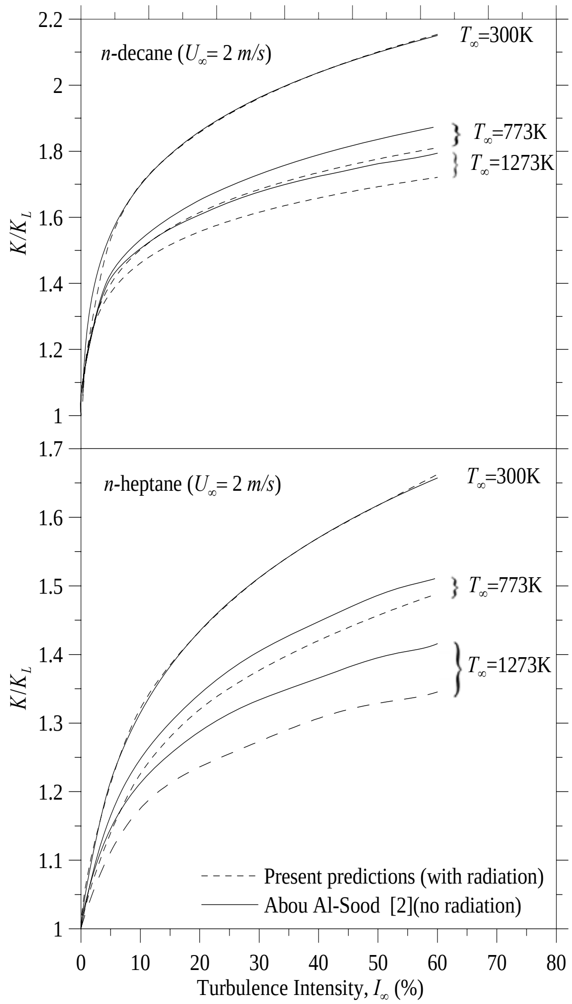

Fig. 4 K/K_L for *n*-heptane and *n*-decane droplets versus turbulence intensity for a typical mean velocity $U_\infty = 2$ *m/s* and various temperatures

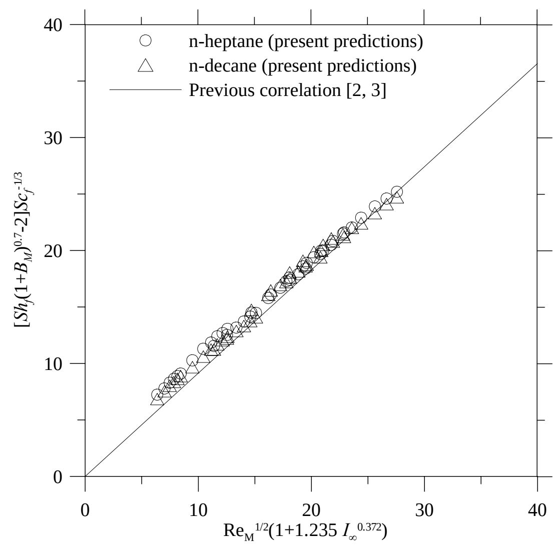

Fig. 5 Comparison of the present predicted turbulent Sherwood number with the one proposed in [2, 3]

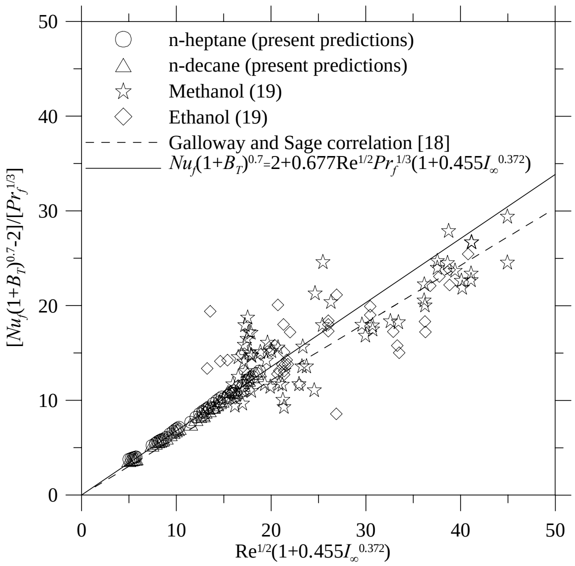

Fig. 6 Predicted turbulent Nusselt numbers for *n*-heptane and *n*-decane versus Reynolds number and its comparison with published data

$$Nu_f(1+B_T)^{0.7} = 2 + 0.677\,\mathrm{Re}_M^{1/2}\,\mathrm{Pr}_f^{1/3}\left(1+0.455 I_\infty^{0.372}\right) \quad (12)$$

This correlation, contains the predicted data for *n*-heptane and *n*-decane droplets at ambient temperatures in the range between 300K and 1273K, freestream mean velocity of 0.6 m/s, 1 m/s and 2 m/s, and turbulence intensity ranging between 0 and 60%. Figure 6 reveals that the present correlation is in a reasonable agreement with the correlation of Galloway and Sage [18], as well as with the experimental data of Yearling [19], although Yearling's data are quite scatted around the proposed correlation, and Galloway and Sage's correlation deviates slightly at high turbulent coefficient, C_T.

Conclusions

A three-dimensional numerical model is developed to assess the effect of freestream turbulence intensity on heat and mass transfer of a droplet exposed to a turbulent, hot airstream. The ambient flow conditions and liquid properties are varied to provide a wide range of test conditions. The major findings of the present study are as follows. The effect of freestream turbulence on droplet evaporation is apparent even at very high ambient temperatures, although it weakens when compared to low and moderate turbulence intensity levels. In addition, the effect of radiation which is negligible at relatively low ambient temperatures becomes significant as the freestream temperature approaches approximately 1000ºC. Also, radiation is more pronounced for the less volatile fuel. The effect of radiation on the previously proposed droplet mass transfer correlation is assessed and found that radiation causes a slight deviation (i.e. less that 5%), which occurs especially at low Reynolds numbers (i.e. at relatively high temperatures). Finally, the effect of turbulence on droplet heat transfer is correlated in terms of Nusselt number. A comparison of the present droplet heat transfer correlation shows a satisfactory agreement with published data.

Acknowledgements

The financial support for this research is provided by the Natural Sciences and Engineering Research council of Canada (NSERC).

References

1. M. Birouk and I. Gökalp, Current status of droplet evaporation in turbulent flows, *Progress in Energy and combustion*, vol. 32, no. 4, pp. 408-423, 2006.
2. M. M. Abou Al-Sood, A numerical study of a droplet evaporating in a turbulent airflow, Ph.D. thesis, University of Manitoba, Manitoba, Canada, 2007.
3. M. M. Abou Al-Sood and Birouk, M., A numerical study of the effect of turbulence on mass transfer from a single fuel droplet evaporating in a hot convective flow, *International Journal of Thermal Sciences*, Vol. 46, pp. 779-789, 2007.
4. I. Gökalp, C. Chauveau, O. Simon and X. Chesneau, Mass transfer from liquid fuel droplets in turbulent flow, *Combust. Flame*, vol. 89, pp. 286-298, 1992.
5. J–S. Wu, Y–J. Lin and H–J. Sheen, Effects of Ambient Turbulence and Fuel Properties on the Evaporation Rate of single Droplets, *Int. J. Heat Mass Transfer*, vol. 44, pp.4593-4603, 2001.
6. J–S. Wu, K–H. Hsu, P-M. Kuo and H–J. Sheen, Evaporation model of a single hydrocarbon fuel droplet due to ambient turbulence at intermediate Reynolds numbers, *Int. J. Heat Mass Transfer*, vol. 46, pp. 4741-4745, 2003.
7. F. R. Menter, Two-equation eddy-viscosity turbulence models for engineering applications, *AIAA Journal*, vol. 32 (8), pp. 1598-1604, 1994.
8. M. M. Abou Al-Sood and M. Birouk, A numerical model for calculating the vaporization rate of a fuel droplet exposed to a convective turbulent airflow, *Int. J. Num. Meth. Heat Fluid Flow* (in press).
9. C. C. Tseng, R. Viskanta, Effect of radiation absorption on fuel droplet evaporation, *Combust. Sci., Technol.* 177(8)(2005) 1511-1542.
10. L. D. Karel, Recent experience with deferent turbulence models applied to the calculation of flow over aircraft components, *Prog. Aerospace Sci.*, vol. 34, pp. 481-541, 1998.
11. S. V. Patankar, Numerical Heat Transfer and Fluid Flow, Hemisphere Publishing Corporation, 1980.
12. J. P. Van Doormall and G. D. Raithby, Enhancement of the simple method for predicting incompressible fluid flows, *Numerical Heat Transfer*, vol. 7, pp. 147-163, 1984.
13. C. M. Megaridis, Comparison between experimental measurements and numerical predictions of internal temperature distributions of a droplet vaporizing under high-temperature

convective conditions, *Combust. Flame*, vol. 93, pp. 287-302, 1993.
14. S.-C. Wong and A.-C. Lin, Internal temperature distributions of droplets vaporizing in high-temperature convective flows, *J. fluid Mech*., vol. 237, pp. 671-687, 1992.
15. S. S. Sazhin, W. A. Abdelghaffar, E. M. Sazhina and M. R. Heikal, Models for droplet transient heating: Effects on droplet evaporation, ignition, and break-up, *Int. J. of thermal Sciences*, vol. 44, pp. 610-622, 2005.
16. B. Abramzon, S. Sazhin, Droplet vaporization model in the presence of thermal radiation, Int. J. Heat Mass Transfer, vol. 48 (9), pp. 1868-1873, 2005.
17. M. M. Abou Al-Sood and Birouk, M., Droplet heat and mass transfer in a turbulent hot airstream. *Int. J. Heat Mass Transfer* (Submitted).
18. T. R. Galloway and B. H. Sage, Thermal and material transport from spheres. Prediction of macroscopic thermal and material transport, *Int. J. Heat Mass Transfer*, vol. 10, pp. 1195-1210, 1967.
19. P. R. Yearling, Experimental determination of convective heat and mass transfer rates from single evaporating droplets in a turbulent air flow, Ph.D. thesis, North Carolina State University, USA, 1995.