Theoretical considerations on ultrasound assisted atomization of fluid sheets

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Abstract

We present theoretical investigations about the mechanism of ultrasound assisted atomization of liquid sheets. Therefore a linear stability analysis of a free fluid sheet influenced by an external, harmonically modulated excitation, in the presence of a gaseous atmosphere, is conducted. The treatment of the hydrodynamic problem for Euler fluids shows that the temporal evolution of the free sheet surfaces can be described by a set of two coupled differential equations with time dependent coefficients. To reach physical relevance and obtain a more realistic fluid characterization, dissipation effects are included. The derived equations were analyzed both analytically by means of the multiple scale perturbation method and by numerical calculations.

Introduction

Atomization, the disintegration of a bulk liquid into fine, preferably monodisperse droplets and the associated mechanisms of surface magnification, is widely used for the combustion in liquid fuel rocket propulsions, diesel engines or gas turbines. It is also used in many industrial applications like spray drying, metall powder, mirco- and nano-particle production or spray coating as well as to produce and deliver agent loaded aerosoles for medical or agricultural usage.

The break-up mechanisms and disintegration processes of liquid jets and sheets usually build up by ducts, narrow slits or impinging jets have been analysed i.e. by Taylor [11], Li & Tankin [6], Lin [7] and several other researchers. As a result, the basic break-up mechanism and thus the droplet sizes are inherently dependent both on the fluid properties and the experimental given flow rates and geometrical specifications. This narrows for a present experimental setup the possibilities on flexible control of the droplet diameter only on variation of the liquid or gaseous flow rates.

Hence ultrasound devices are used to affect the disintegration process independent of gaseous or liquid flow rates by means of an external excitation. Several experimental investigations as well as numerical aproaches using CFD (see for example [1], [10], [5]), have been conducted showing that a destabilization of oscillatory wave modes on the liquid sheet surfaces is excited, which initialize sheet break up processes, due to high frequency forcing. And thus, the formation of ligaments and droplets is directly influenceable by an external excitation.

In spite of the wide variety of applications and advantages of these atomization method, the basic physical mechanism and principles of the break-up process have not been investigated sufficiently up to now. In order to have a better understanding of the ultrasound assisted fluid atomization, the stability of a free liquid sheet in the presence of a harmonic modulated external forcing is investigated analytically.

The Hydrodynamic System

We consider a layer of a ideal, immiscible and incompressible fluid with an undisturbed thickness $H = 2h$ which is surrounded by an assumed ideal, immiscible and incompressible gaseous atmosphere. A surface tension σ act between the fluid layer and the gaseous ambiente. The fluids were externally forced due to high frequency time variing gravitational or pressure fields in direction perpendicular to the fluid interfaces. Fig. 1., 2. show the schematic setup of the considered problem.
Starting from the basic hydrodynamic equations, the dynamic of the fluid bulk is governed by

$$\rho_i[\partial_t + \mathbf{u} \cdot \nabla]\mathbf{u}_i = -\nabla\Pi_i(\mathbf{x}, t) \qquad (1)$$

where the velocity field $\mathbf{u}$ have to fullfill the continuity equation

$$\nabla\mathbf{u}_i = 0. \qquad (2)$$

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Proceedings of the 21^{th} ILASS-Europe Meeting 2007

The subscript $i = 1, 2$ denotes the respective liquid or gaseous fluid layers. The value ρ_i represents the density of the fluids, where Π denotes body forces generated by a external, time variing gravitational or pressure field. Below, we don' t have to distinguish between modulated pressure or gravity fields, because both act as volumina forces. Thus we choose without loss of generality a time variing gravity field, whereas the case of a modulated pressure field can be treaten in the same way.

The dynamic of the interfaces $\eta_j(\mathbf{x}, t); \quad j = 1, 2$, which seperate the fluid layers, is governed by the kinematic surface condition

$$\frac{D}{Dt}\eta_j(\mathbf{x}, t) = \partial_t \eta_j + \mathbf{u} \cdot \nabla \eta_j = 0. \qquad (3)$$

We eliminate the horizontal velocity in Eq.(1) by multiplying with $\mathbf{e_z} \cdot \nabla \times \nabla \times$ and performing a transformation into a frame of reference that moves with the external forcing. Thus in Eq. (1) the explicite time dependence drops out.

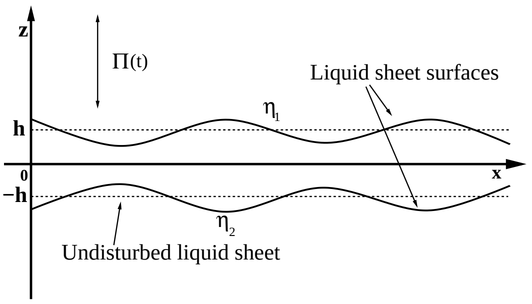

Figure 1. Sketch of a symmetrical wave mode. Initially the liquid layer has a undisturbed thickness $H = 2h$, whereas the surfaces are located at $z = \pm h$.

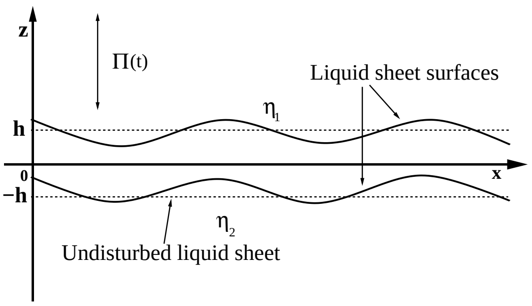

Figure 2. Antisymmetrical wave mode.

Boundary Conditions

Assuming that the fluid layers initially are stationary and seperated by flat interfaces, we have to regard several interfacial boundary conditions. Introducing the notation

$$[\![X]\!] \equiv X_i - X_j|_{\eta_k} \qquad (4)$$

which denotes the jump of the value X at the k-th interface seperating the fluids i and j, at the interfacial boundaries the velocity has to be continuous

$$[\![\mathbf{u}]\!] = 0 \qquad (5)$$

and the jumps of the normal stresses are balanced by surface tension

$$[\![\rho \partial_t \mathbf{u}]\!] + [\![\rho f W(t)]\!] = -\sigma k^4 \eta. \qquad (6)$$

Here we have to remark that the boundary conditions for the normal stresses become explicitly time dependent, due to the transformation into a new frame of reference.

Linear Evolution Equations

In the comoving frame of reference we have initially no fluid movement, a constant hydrostatic pressure and interfaces between the fluid layers with a plane geometry. In order to analyse the stability of the basic state, we linearize the hydrodynamic equations as well as the boundary conditions around the basic solutions $\mathbf{u} = \eta_i = 0$. It can be shown that after linearization the horizontal and vertical flow fields decouple. In consequence of the fact that in linear regime the temporal evolution of the fluid interfaces (3) is only affected by the vertical flow field, we can separate flows and consider only fluid movements in z-direction:

$$\begin{aligned} \partial_t(\partial_{zz} - k^2)w_1 &= 0, h \le z \le \infty && (7)\\ \partial_t(\partial_{zz} - k^2)w_2 &= 0, -h \le z \le h && (8)\\ \partial_t(\partial_{zz} - k^2)w_3 &= 0, -\infty \le z \le -h, && (9) \end{aligned}$$

where w_i is the vertical velocity in the fluid layer i and k the wave number. Due to the requirement that the surrounding fluid layers are of infinit high, it is assumed that the velocity field in the outer sheets tends to zero for $z \to \pm\infty$, thus the solution of Eq. (7) - (9) can be expressed as

$$\begin{aligned} w_1(z, t) &= A(t) \exp(-kz) && (10)\\ w_2(z, t) &= B(t) \sinh(kz) + C(t) \cosh(kz) && (11)\\ w_3(z, t) &= D(t) \exp(kz). && (12) \end{aligned}$$

After determine the unknown time dependent functions $A(t), B(t), C(t), D(t)$ by means of the linearized interfacial boundary conditions (5), (6) and kinetic surface condition (3) we obtain, introducing symmetric and antisymmetric modes of the interface deformations, denoted as $\eta^s(t)$, $\eta^a(t)$ and given by

$$\begin{aligned} \eta^s(t) &= \eta_1(t) - \eta_2(t) && (13)\\ \eta^a(t) &= \eta_1(t) + \eta_2(t), && (14) \end{aligned}$$

a system of coupled differential equations with time variing coefficients:

$$\partial_t^2 \eta^s(t) = -T_k^s \left[k^2 \sigma \eta^s(t) - \tilde{\rho} f W(t) \eta^a(t)\right] \quad (15)$$
$$\partial_t^2 \eta^a(t) = -T_k^a \left[k^2 \sigma \eta^a(t) - \tilde{\rho} f W(t) \eta^s(t)\right], \quad (16)$$

where we use $T^a = k/\coth(kh), T_k^s = k \coth(kh)$ and $\tilde{\rho} = \rho_1 - \rho_2$ as abbreviations.

Phenomenological Incorporation of Viscous Damping

The movement of a real fluid leads always to energy dissipation due to the fluids viscosity. In cases of a small viscous dissipation and under the condition that the velocities are not too large, one can calculate a damping coefficient [4] by considering the ratios of the averaged kinetic energy $\overline{E_{kin}}$ and the associated averaged temporal dissipation rate $\overline{\partial_t E_{kin}}$, where

$$\partial_t E_{kin} = -2\rho_i \nu_i \int (\nabla_i \mathbf{u}_j)^2 dV \quad (17)$$

and

$$E_{kin} = \rho_i \int \mathbf{u}^2 dV \quad (18)$$

hold. By means of the solutions (10) - (12) a damping coefficient can be determined

$$\gamma = \frac{1}{2} \frac{\overline{\partial_t E_{kin}}}{\overline{E_{kin}}} = 2k^2 \nu. \quad (19)$$

To simplify matters, we have only considered the properties of the fluid sheet, because its density as well as its viscosity is at least up to orders of magnitude higher than the values of the gaseous atmosphere and hence have a bigger influence on the damping. After incorporation of the derived damping coefficient in the differential equation system (15), (16) the temporal evolution equations for external forced weak viscous fluids read

$$\begin{aligned}(\partial_t \; + \; 2\gamma^a)\partial_t \eta^a = & \quad (20)\\ - \; T_k^a \left[-\alpha \eta^a k^2 - \tilde{\rho} g (1 + f W(t)) \eta^s\right] & \\ (\partial_t \; + \; 2\gamma^s)\partial_t \eta^s = & \quad (21)\\ - \; T_k^s \left[-\alpha \eta^s k^2 - \tilde{\rho} g (1 + f W(t)) \eta^a\right] &\end{aligned}$$

Analytical treatment

The derived evolution equations (15), (16) as well as (20), (22) can not be solved in general. Thus a approximate analytical method is used. Assuming that the amplitude of the external forcing is sufficient small, we follow Nayfeh and Mook [9] and Mohammed et al. [8], to obtain asymptotic expressions to determine the stability of the given problem, using the method of multiple time scales.

Introducing a smallness parameter ϵ, we can define different time scales given by

$$T_n = \epsilon^n t, \quad n = 0, 1, 2..., \quad (22)$$

where the time derivative operators can be expressed as

$$\frac{d}{dt} = D_0 + \epsilon D_1 + ...; \quad \frac{d^2}{dt^2} = D_0^2 + \epsilon D_0 D_1 + ... \quad (23)$$

and $D_l^k = \partial^k / \partial T_l^k$ hold.

Rewriting equations (15), (16) as

$$(\partial_t^2 + \alpha_{11})\eta^a + (\alpha_{21} + \epsilon f_{12} W(t))\eta^s = 0 \quad (24)$$
$$(\partial_t^2 + \alpha_{22})\eta^s + (\alpha_{12} + \epsilon f_{21} W(t))\eta^a = 0 \quad (25)$$

and assume that the solutions of the system (24), (25) may be expanded in the form

$$\eta^{a,s}(t, \epsilon) = \eta_0^{a,s}(T_0, T_1) + \epsilon \eta_1^{a,s}(T_0, T_1) + \ldots \quad (26)$$

we substitute (26), (22) and (23), into (24), (25) and equating like powers of ϵ. Thus we find in lowest order

$$(D_0^2 + \alpha_{11})\eta_0^a + \alpha_{12}\eta_0^s = 0 \quad (27)$$
$$(D_0^2 + \alpha_{22})\eta_0^s + \alpha_{21}\eta_0^a = 0. \quad (28)$$

The solutions of (27), (28) can be sought in the form

$$\eta_0^a = \sum_j^2 \alpha_{12} A_j(T_1) e^{i\omega_j T_0} + cc \quad (29)$$

$$\eta_0^s = \sum_j^2 (\omega_j^2 - \alpha_{12}) A_j(T_1) e^{i\omega_j T_0} + cc, \quad (30)$$

where the unknown A_j is a complex, slow variing amplitude and cc is the abbreviation of the complex conjugate expressions of the preceding terms. For solvability the frequencies $\omega_j, \quad j = 1, 2$ have to fullfill the relation

$$\omega_1^2 = \frac{\alpha_{11} + \alpha_{22}}{2} + \frac{\left[(\alpha_{11} - \alpha_{22})^2 + 4\alpha_{12}\alpha_{21}\right]^{\frac{1}{2}}}{2}, \quad (31)$$

and

$$\omega_2^2 = \frac{\alpha_{11} + \alpha_{22}}{2} - \frac{\left[(\alpha_{11} - \alpha_{22})^2 + 4\alpha_{12}\alpha_{21}\right]^{\frac{1}{2}}}{2}. \quad (32)$$

Substituting the solutions (29), (30) into the first order expansion of the system, given by

$$\begin{aligned}(D_0^2 + \alpha_{11})\eta_1^a + \alpha_{12}\eta_1^s \; = & \quad (33)\\ -2D_0 D_1 \eta_0^a \; + \; f_{12} W(T_0) \eta_0^s & \\ (D_0^2 + \alpha_{22})\eta_1^s + \alpha_{21}\eta_1^a \; = & \quad (34)\\ -2D_0 D_1 \eta_0^s \; + \; f_{21} W(T_0) \eta_0^a, &\end{aligned}$$

we choose

$$\eta_1^a = \sum_j^2 \alpha_{12} B_j(T_1) e^{i\omega_j T_0} + cc \qquad (35)$$

$$\eta_1^s = \sum_j^2 (\omega_j^2 - \alpha_{12}) B_j(T_1) e^{i\omega_j T_0} + cc, \qquad (36)$$

as solutions, due to the structure of the lhs of Eqs. (33), (34). Taking into account the conditions (31), (32) this choice yields to requirements for the complex coefficients A_j of the zero order solution which build the secular terms on the rhs of the first order expansion (33), (34):

$$\sum_j^2 -2iD_1\alpha_{12}\omega_j A_j(T_1)e^{i\omega_i T_0} - \qquad (37)$$
$$f_{12}(\omega_j^2 - \alpha_{11})A_j(T_1)e^{i\omega_i T_0}W(T_0) + cc = 0$$
$$\sum_j^2 -2iD_1(\omega_j^2 - \alpha_{11})A_j(T_1)e^{i\omega_i T_0} - \qquad (38)$$
$$f_{21}\alpha_{12}A_j(T_1)e^{i\omega_i T_0}W(T_0) + cc = 0.$$

As a consequence of the ansatz (35), (36) and depending on the external forcing $W(T_0)$, the secular terms have to vanish.

In order to solve the problem of a high frequency, ultrasonic excitated fluid sheet, we specify the external periodic forcing term as $W(T_0) = \cos(2\Omega T_0)$, and hence the differential equations (15), (16) as well as (20), (22) become coupled equations of the Mathieu-type. We restrict our analytical treatment on the case of main resonance, which appears, if the stimulation frequency Ω approaches ω_1 or ω_2. Treating the case $\Omega \approx \omega_1$, where the second case with $\Omega \approx \omega_2$ could be easily received by means of the substitution $\omega_1 \longrightarrow \omega_2$, we introduce a detuning parameter σ which is given as

$$\epsilon\sigma = \omega_1 - \Omega. \qquad (39)$$

The incorporation of Eq. (39) in Eqs. (37), (37) leads, under the assumption $\omega_2 \gg \omega_1$ and after the neglect of high frequency terms, due to the Rotating Wave Approximation, to relations for the elimination of the secular terms. After some algebraic calculations it can be derived that the slow variing amplitude $A_1(T_1)$ has to fullfill the equation:

$$D_1 A_1 = i\frac{\alpha_{21}f_{21} + f_{12}(\omega_1^2 - \alpha_{11})}{2\omega_1(\omega_1^2 - (\alpha_{11} - \alpha_{12}))}\bar{A}_1 e^{i2\sigma T_1} \qquad (40)$$

where $\bar{A}_1$ denotes the conjugate complex amplitude. In order to solve Eq. (40) we separate A_1 into its real and imaginary part

$$A_1 = (u + iv)\exp(i\sigma T_1). \qquad (41)$$

and substitute (41) into (40). Equating the derived expressions into real and imaginary values, we obtain two differential equations for u and v:

$$D_1 u = \left(\frac{\alpha_{21}f_{21} + f_{12}(\omega_1^2 - \alpha_{11})}{2\omega_1(\omega_1^2 - (\alpha_{11} - \alpha_{12}))} + \sigma\right) v \qquad (42)$$

$$D_1 v = \left(\frac{\alpha_{21}f_{21} + f_{12}(\omega_1^2 - \alpha_{11})}{2\omega_1(\omega_1^2 - (\alpha_{11} - \alpha_{12}))} - \sigma\right) u \qquad (43)$$

Using the ansatz

$$u = \xi\exp(\lambda T_1) \qquad (44)$$
$$v = \zeta\exp(\lambda T_1), \qquad (45)$$

we obtain the amplitude (41)

$$A_1 = (\xi + i\zeta)\exp((\lambda + i\sigma)T_1). \qquad (46)$$

Thus the system is stable if $\lambda < 0$, with

$$\lambda = \pm\sqrt{\left(\frac{(\omega_1^2 - \alpha_{11})f_{12} + \alpha_{12}f_{21}}{4\omega_1(\omega_1^2 - \alpha_{11} + \alpha_{12})}\right)^2 - \sigma^2}. \qquad (47)$$

In the case of critical system behaviour λ has to vanish. This requirement leeds to

$$\sigma_{1,2} = \pm\left(\frac{(\omega_1^2 - \alpha_{11})f_{12} + \alpha_{12}f_{21}}{4\omega_1(\omega_1^2 - \alpha_{11} + \alpha_{12})}\right). \qquad (48)$$

Hence the transition curves are given by the expressions

$$\Omega = \omega_1 + \epsilon\sigma_1(\omega_1) \qquad (49)$$
$$\Omega = \omega_1 + \epsilon\sigma_2(\omega_1). \qquad (50)$$

The stability relations in the case of weak viscous fluid sheets can be derived by asumming a small damping given by $\gamma = \epsilon\Gamma$. This leads to the system

$$(\partial_t^2 + \epsilon 2\Gamma\partial_t + \alpha_{11})\eta^a + (\alpha_{21} + \epsilon f_{12}W(t))\eta^s = 0 \qquad (51)$$
$$(\partial_t^2 + \epsilon 2\Gamma\partial_t + \alpha_{22})\eta^s + (\alpha_{12} + \epsilon f_{21}W(t))\eta^a = 0 \qquad (52)$$

which can be treaten as in the preceding calculations.

Numerical Treatment

To confirm the approximative analytical calculations, we examine the derived evolution equations (15), (16) and (20), (22) by numerical means.
Because we consider a time periodic excitation with period $2\pi/\Omega$ given by $W(t) = \cos(2\Omega t)$, one can assume, as a result of the Floquet Theory, that the solutions of the considered equations are also time harmonic functions, having the form

$$\eta^i(t) = e^{\lambda t}\tilde{\eta}^i(t), \quad \tilde{\eta}^i(t) = \tilde{\eta}^i(t + 2\pi); \quad i = s, a \qquad (53)$$

Following Beyer & Friedrich [3], we expand the periodic functions $\tilde{\eta}^i(t)$ into Fourier series

$$\eta^i(t) = e^{(\lambda + i\mu)t}\lim_{n\to\infty}\sum_{n=-N}^{N}\eta_n^i e^{in\Omega t} \qquad (54)$$

and obtain an infinite dimensional algebraic set of equations for the amplitudes of the wave modes

$$A_n^s \eta_n^s = \alpha_{12}\eta_n^a + \epsilon f_{12} \sum_{n'=-N}^{N} W_{n,n'}\eta_{n'}^a \qquad (55)$$

$$A_n^a \eta_n^a = \alpha_{21}\eta_n^s + \epsilon f_{21} \sum_{n'=-N}^{N} W_{n,n'}\eta_{n'}^s \quad , \qquad (56)$$

where the coefficients A_n^s, A_n^a are given as

$$A_n^s = 2\left[\{\lambda + i(\mu + n\Omega)\}^2 + \alpha_{11}\right] \qquad (57)$$

$$A_n^a = 2\left[\{\lambda + i(\mu + n\Omega)\}^2 + \alpha_{22}\right] \qquad (58)$$

and the excitation terms leads to

$$W_{n,n'} = \frac{1}{2\pi}\int_0^{\frac{2\pi}{\Omega}} d\tau e^{i(n-n')\Omega\tau} W(\tau). \qquad (59)$$

The truncation of the Floquet ansatz at a finit but adequate high value of N leads to a linear, finite-dimensional eigenvalue problem. Further on, to obtain critical behavior of the system, we have to demand $\lambda = 0$ in Eqs. (57), (58) and choose $\mu = 0$ or $\mu = 1/2$ to calculate the stability branches for the harmonic or subharmonic solutions, respectively. Applying this we can numerical solve system (55), (56) and determine the critical forcing amplitude $\epsilon = f_c$ by means of standart routines.

Results and Discussion

With the analytical derived expressions (49), (50) we are able to calculate the instability branches for a free liquid sheet under an harmonic high frequency, ultrasonic excitation. For the subsequent considerations we have used the fluid parameters of water to model the fluid sheet and the values of air for the surrounding gaseous atmosphere. A thickness of $h = 1e^{-3}$ cm for the fluid sheet was selected and a forcing frequency was choosen as $\Omega/2\pi = 50$ kHz. Figure 3. shows the results of the multiple time scale approximation.

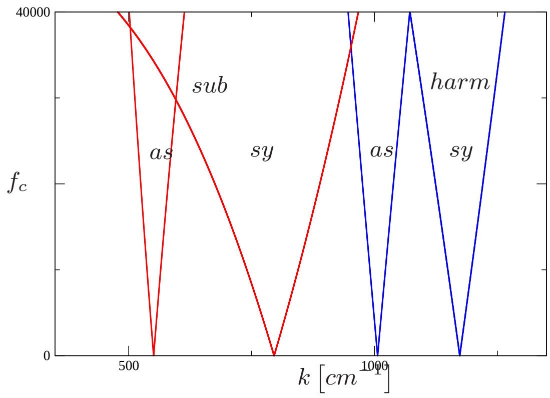

Figure 3. Analytically calculated stability regions of an ideal fluid sheet. f_c denotes the scaled critical forcing amplitude and k the wave number in $[cm^{-1}]$. Transition curves of the subharmonic solutions are termed as "sub" whereas the branches of the harmonic solution are denoted by "harm". The substructures belonging to the antisymmetric "as" or symmetric "sy" wave modes respectively.

As in the case of the classical Faraday Instability [2], the (f_c, k)-plane is seperated in different tongue-like regions, belonging to the subharmonic and harmonic solutions. As a result it can be seen, that the harmonic as well as the subharmonic tongue has a substructure. The solvability relations (31), (32) allow to distinguish between tongues belonging to the antisymmetrical and the symmetrical wave modes, respectively.

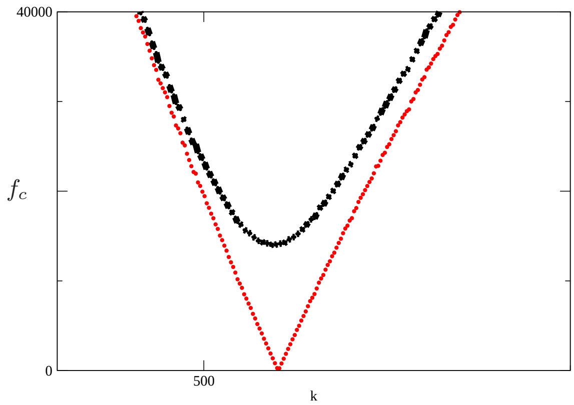

Figure 4. Influence of damping on the stability regions of a fluid sheet. The transition curve with red dots (•) belongs to the undamped fluid sheet whereas the curve with black crosses (+) belongs to the subharmonic solution of antisymmetrical wave mode of the the damped system.

Due to the neglect of viscosity, we see that the tips of the instability zones reach the axis of abscissae. This means in the physical interpretation, that infinitesimal small forcing amplitudes can destabilize the fluid sheet and thus initialize sheet break up. This quite unphysical result was revised due to the phenomenological modelling of viscous dissipation effects. The incorporation of damping terms into the system leads to a excitation threshold as shown in Fig. 4. and thus a finite (critical) forcing amplitude is necessary to excite waves on the fluid sheet.

Figures 5. and 6. show the numerical calculated stability regions. The obtained results confirm the analytical considerations. Furthermore one can see that the harmonic solutions are in general more damped then the subharmonic ones, whereas excitation of symmetric wave modes need a higher forcing amplitude compared to the antisymmetric wave modes. As a result subharmonic, asymmetric waves are the most unstable, because their critical threshold is the lowest one.

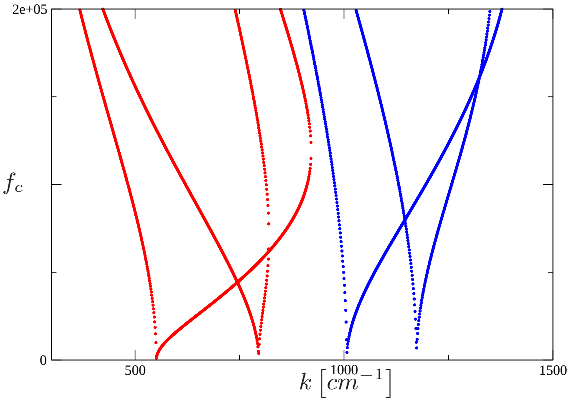

Figure 5. Numerical calculated stability regions of an undamped liquid sheet

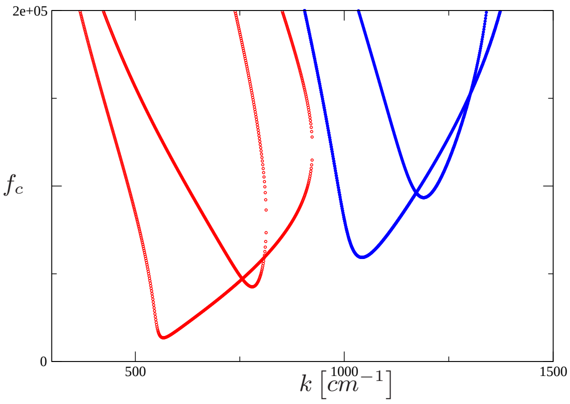

Figure 6. Influence of damping on the stability regions of a fluid sheet. The critical wave number k_c belongs to the minima of the tongues

It has to be pointed out, that the interpretation of the subtongues as instability regions belonging to the antisymmetric and symmetric wave modes was only possible by means of the analytical calculations.

Conclusion

We have derived the basic equations describing the external excitation of the surfaces waves of a free liquid sheet in a gaseous atmosphere. Thereby we have shown that the linear stability analysis leads to a set of coupled differental equations with time dependent coefficients. The equations have been extended, due to the inclution of viscous dissipation effects. Considering an external excitation in the form $W(t) = \cos(2\Omega t)$, the differential equations system becomes coupled equations of the Mathieu-type. These equations have been solved analytical as well as numerical. The results of the calculations show that the instability zones for a thin liquid sheets under an external harmonic forcing splits into branches for the symmetric and antisymmetric surface wave modes for subharmonic and harmonic solutions respectively. This result shows that the principal mechanism of ultrasonic assisted fluid atomization is given by parametric resonance.

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