ABSTRACT In this study, experiments were conducted to investigate the breakup characteristics of mono-dispersed diesel droplets in a cross-flow air stream. And the numerical analysis was also implemented using the mathematical models such as TAB (Taylor Analogy Breakup), and DDB (Droplets Deformation and Breakup) models. In order to analyze the droplet breakup phenomena, the regimes of breakup are investigated by using the droplet visualization system composed of high speed droplet generator, a long distance microscope system with the optical and spark lamp system.

The atomization characteristics of mono-dispersed droplets and the distribution of droplet size in both radial and axial directions were obtained to analyze the breakup characteristics according to the Weber number based on the relative velocity between a droplet and an air stream. In addition, the accuracy of widely used TAB and DDB models was verified comparing to the experimental results, such as the deformation rate and trajectory of droplet before breakup, and the droplet size distribution after breakup. In order to supplement the prediction accuracy of the mathematical models, additional attempts of the modification of drag model, and adopting the Rosin-Rammler size distribution function, to fit with the experimental results were performed and its results were also compared to the those of the drag model and size distribution function included in the original KIVA code.

Keywords: Droplet breakup, Breakup models, Size distribution, Monodispersed-droplet

1. INTRODUCTION

In a direct injection Diesel engine, the atomization performance of diesel fuel is much important because of its close relation to the engine efficiency and pollutant emissions. That is, the efficiency of combustion is strongly influenced by fuel vaporization and the vaporization is much dependent on the atomization of fuel spray since the surface area exposed to ambient gas becomes larger with the active atomization process. In particular, understanding the breakup characteristics of mono-dispersed droplets in gas flow field is the essential job in developing a highly efficient diesel engine because the droplet breakup is a foundation of atomization of fuel spray.

In order to find out the breakup characteristics of droplet breakup, a lot of theoretical and experimental researches have been performed for many years. Reviews of liquid atomization mechanisms have been provided by Krzeczkowski [1], Liu and Reitz [2], and Lee and Reitz [3]. In accordance with the previous studies, droplet experiences two kinds of breakup stage, i.e. first breakup stage and second breakup stage, with the increase of relative velocity between a droplet and ambient gas. During the first breakup stage, when a liquid droplet is exposed to steady gas flow field, the droplet is deformed without disintegration. As the relative velocity is more increased, three breakup regimes are encountered, for example, bag breakup (Kenndy and Roberts [4]), ‘shear’ or ‘boundary layer stripping’ breakup (Ranger and Nicholls [5]), and catastrophic breakup (Reinecke and Waldman, [6]). Liu and Reitz [7], especially, have pointed out that the widely referred to shear or boundary layer stripping mechanisms are not consistent with their experimental results. Instead, they suggest that this regime should be referred to as a ‘stretching and thinning’ breakup regime instead of a ‘boundary layer stripping’ regime. In addition, some criteria for predicting droplet breakup regime transitions based on the Weber number, $W_e = \frac{\rho_g U^2 d}{\sigma}$ (where $\rho_g$ is the gas density, $U$ is the relative velocity between the droplet and the ambient gas, $d$ is the droplet diameter, and $\sigma$ is the surface tension) have been presented by Krzeczkowski [1], Wu and Faeth [8], and other authors.

In order to reveal out the spray performance of liquid fuel, some experimental apparatuses can be used. For example, the PDPA (phase Doppler particle analyzer) system is widely used for obtaining the droplet size and velocity. However there are some limitations on understanding the breakup characteristics of liquid droplet in detail. In order to investigate the droplet breakup mechanisms, the long-distance microscope and single droplet generator can be utilized. By analyzing the photos of liquid droplet shadowgraphs, the deformation and breakup regimes can be obtained. Lee and Reitz [3] investigate the effect of liquid properties on the distortion and breakup mechanisms of liquid droplets and show that the breakup mechanisms of droplets with the different liquids were similar at atmospheric and elevated ambient pressure conditions provided to maintain the Weber number at constant. And Hwang et al. [9] experimentally and numerically approach the droplet breakup phenomena and show the measured and predicted deformation and trajectory of liquid droplet in a gas flow field.

In addition, modeling of the atomization process of high-speed diesel spray is the hot issue in developing a internal combustion engine, gas turbine, and industrial furnace because the accurate prediction of atomization gives us precise predictions of evaporation, combustion and
emissions formation. In many years, a lot of numerical models of fuel spray have been developed such as mathematical models and empirical models. In particular, TAB (Taylor Analogy Breakup) model and DDB (Droplet Deformation and Breakup) model are the most widely used mathematical numerical models. The TAB models, which is included in the original KIVA code, is suggested by O’Rourke and Amsden [10] by assumption of the analogy between the distorting droplet and the vibrating spring-mass system. In comparison to the TAB model, Ibrahim [11] proposed the DDB model which is assumed that the droplet is deformed owing to a pure extensional flow. Additionally, a number of breakup models for primary breakup based on the wave instability, such as Kelvin-Helmholtz wave instability (KH) model, and Rayleigh-Taylor instability (RT) model have been developed by many researchers. Also, for the analysis of both primary and secondary breakup, the hybrid breakup models that two different breakup models are combined have been widely used.

In this work, since the only droplet which experiences a breakup process was considered without a liquid jet breakup, the TAB and the DDB models were used. Using these models, and KIVA code, the droplet deformation rate, its trajectory in first breakup stage, and the droplet size distribution in second breakup stage were estimated and also assessed the prediction ability of the two models. In particular, in the condition of cross-flow field, there are some differences between the experimental results and those of the numerical models. Accordingly, the aim of this study is to find out the optimum droplet drag model, and droplet size distribution function in the KIVA code.

2. EXPERIMENTAL APPARATUS AND PROCEDURE

2.1 Experimental Apparatus

In order to observe the droplet breakup phenomena in a cross-flow field, the experimental apparatus consisted of a single droplet generator, an air flow nozzle, spark lamp, and a long distance microscope equipped with a digital camera as shown in Fig. 1. The mono-dispersed droplets are generated using an vibrating orifice in a droplet generator, and then the droplets whose diameter is 189 µm are exposed to the air flow field made by the air nozzle. The pressure difference of droplet generator was set constant to 0.1 MPa in gauge pressure. Diameter of the nozzle exit is 2 mm and the distance between the nozzle exit and droplets passage was maintained constant to 1.5 mm as illustrated in Fig. 2.

The PDPA system was also utilized to find out the distribution of droplet size according to the Weber number. The effective range of PDPA signal analyzer, was from 4 µm to 200 µm, with consideration of droplet size of 189 µm produced by the droplet generator.

2.2 Experimental Procedure

In order to observe the droplet breakup phenomena in a cross-flow field, the mono-dispersed droplets were generated using a vibrating orifice of the droplet generator and injected into the air flow stream ejected from the air nozzle. The velocity of the air jet was determined by controlling the flow meter. Using the spark lamp, long-distance microscope and digital camera, which are illustrated in Fig. 1, the shadowgraphs of droplets that experience the breakup process were obtained. In this experiment, deformation rate of a distorting droplet and the droplet’s trajectory according to the Weber number were measured by the analysis of the shadowgraphs.

Especially, the droplet shape change is the peculiar characteristic in the first breakup stage. Therefore, for analyzing the special characteristics, the deformation rate was defined as the ratio of the major axis of the deformed droplet to the diameter of the intact droplet. In addition, for obtaining the droplet’s trajectory, the dropping and traveling distances of the droplet exposed to the air jet were measured. Because of the difficulty when defining the droplet’s path after breakup, the trajectory was verified only in the first breakup stage.

Once a droplet is broken, a number of smaller droplets are newly generated and distributed near the droplet’s path. In the macroscopic view point, therefore, plenty of droplet size will be found out in both radial and axial directions. Accordingly, the PDPA measurement points for obtaining the droplet size were specified at each breakup regimes based on the macroscopic image shown in Fig. 3. As can be seen in Fig. 3, the measurement points were arranged at the intervals of 2 mm in both radial and axial directions within the range in which the droplets are clearly viewed. Additionally, the origins of the Z and R coordinates were the passage of the mono-dispersed droplet and the center of the air jet nozzle, respectively, as shown in Fig. 3 (a).

3. NUMERICAL MODELING

3.1 The TAB model
Taylor Analogy Breakup (TAB) model that is suggested by O’Rourke and Amsden [10] is based on the analogy between a distorting droplet and a spring-mass system as shown in Fig. 4. According to the Taylor analogy, the restoring force of the spring is analogous to the surface tension forces and the external force on the mass is analogous to the gas aerodynamic force. And it is also supposed that the damping force on the droplet is due to the viscosity. Defining the dimensionless displacement of the equator of the droplet from its equilibrium position,

\[ y = y_c / r_c \]

the governing 2nd order differential equation of the oscillating droplet is given by,

\[ \ddot{y} = \frac{C_p \rho_L r^2}{C_d \rho_p r^2} - \frac{C_s \sigma}{\rho_p r^2} y - \frac{C_d \mu_c}{\rho_p r^2} \dot{y} \]  

In Eq. (1), \( r \) is the radius of the intact droplet and subscripts, \( L \) and \( G \) indicate the diesel fuel and the air, respectively. The analytic solution of the Eq. (1) can be obtained manually because of its linearity. In addition, the breakup criterion of the TAB model is postulated to be occurred when the dimensionless displacement, \( y \) becomes larger than 1.0 at any conditions of Weber number.

3.2 The DDB model

Ibrahim [11] proposed the ‘Droplet Deformation and Breakup (DDB)’ model that the droplet is deformed due to a pure extensional flow from an initial spherical shape. In this model, since all forces are assumed to act through the center of mass (c.m.) of the half droplet, the distance \( y_{c.m.} \) from the c.m. of the deforming half droplet (Fig. 5) was introduced in developing the governing equation. From these considerations, the DDB model includes the nonlinear effects which are not considered in the TAB model. Its governing equation is derived from the energy equation of the half droplet and given by,

\[ K \frac{d^2 y^*}{dt^2} + \frac{4N}{\text{Re}_G y^*} \frac{dy^*}{dt} + \frac{27\pi}{16 \text{We}} y^* \left[ 1 - 2(cy^*)^{-6} \right] = \frac{3}{8} \]  

where \( y^* = y_{c.m.} / r \) is the dimensionless distance of the c.m., \( K \) and \( N \) are the ratios of liquid-gas density and viscosity, respectively. Due to the nonlinearity of the equation, its solution can’t be obtained manually. Therefore its solution was obtained by using the Runge-Kutta method. On the opposite side to the case of TAB model’s fixed breakup criterion, the deformation of the droplet, when the breakup occurs, is proportional to the Weber number as illustrated in Eq. (3).

\[ \frac{a}{r} = \frac{\text{We}}{6\pi} \]  

where \( a \) is the major semi-axis of the ellipsoidal cross-section of the oblate spheroid.

Fig. 3. Measurement points of the PDPA experiment according to the breakup regimes

Fig. 4. Analogy between the distorting droplet and the spring–mass system

Fig. 5. The deforming half-droplet of DDB model
3.3 The droplet drag model
When a droplet enters into the air stream which flows downstream, the aerodynamic force pressures the droplet to be deformed and simultaneously fall down. Therefore, the droplets travel through a parabolic orbit. In the consideration of droplet trajectory, Liu et al. [12] determined a droplet’s acceleration from,

\[ \rho_g V_d \frac{d\mathbf{u}}{dt} = C_D \rho_g \frac{U^2}{2} \left( \mathbf{U} - \mathbf{U} \right) \]

where \( \mathbf{u} \) is the droplet velocity vector, \( \mathbf{U} \) is the droplet-air relative velocity vector, \( C_D \) is the droplet drag coefficient, \( V_d = 4\pi^3 / 3 \) and \( A_f = \pi r^2 \) are the volume and frontal area, respectively (for a spherical droplet). The droplet drag coefficient is usually given by that of a rigid sphere as shown below.

\[ C_{D,S} = \frac{24}{Re} \left( 1 + \frac{1}{6} Re^{2/3} \right) \quad Re \leq 1000 \]
\[ C_{D,S} = 0.424 \quad Re > 1000 \]

Because of the fact that the drag coefficient of a distorting droplet should lie between that of a rigid sphere and that of a disk, Liu et al. [12] suggested the droplet drag coefficient related that of a rigid sphere as follow:

\[ C_D = C_{D,S} (1 + 2.632 y) \]

Using the KIVA code that originally includes the above equation as the droplet drag model, the dropping distance of a droplet is much underestimated in comparison to the results of experiment. This illustrated that the drag coefficient determined by Eq. (6) is much smaller than that of the practice. Hence, Liu and Reitz [7] used the DDB model with \( A_f = \pi a^2 \) to compute the droplet frontal area.

In the present study, additionally, the effect of the variance of the droplet frontal area was adopted to both the TAB and the DDB. Therefore, the formulation of Eq. (6) the KIVA code was modified as follow:

\[ C_D = C_{D,S} (1 + 2.632 y)(1 + y + 0.25 y^2) \]

In the above equation, the second parenthesis expresses \( (a/r)^2 = (1 + 0.5 y)^2 \) And, in the case of the DDB model, the relation between \( y \) of the TAB and \( y^* \) of the DDB in Eq. (8) was adopted and \( y^* \) was set to 2 when its value becomes larger than two.

\[ y = 2 \left( \frac{3}{4} \pi y^* - 1 \right) \]

3.4 The droplet size distribution function
In the present study, the Rosin-Rammler size distribution function is additionally considered to compare the prediction accuracy with that of the original KIVA code. The Rosin-Rammler cumulative distribution function (CDF) is:

\[ CDF = \int_0^D f(D) dD = 1 - \exp \left[ - \left( \frac{D}{X} \right)^q \right] \]

where,

\[ f(D) = \frac{q D^{q-1}}{X^q} \exp \left[ - \left( \frac{D}{X} \right)^q \right] \]

so that the droplet diameter is obtained using a uniformly distributed random number, RN, with

\[ D = X \left[ \ln(1 - RN)^{-1} \right]^{1/q} \]

In Eq. (11), \( X \) represents characteristics droplet size and \( q \) is an empirical dimensionless constant to specify the distribution width. In the present study, \( X \) corresponds to the Sauter mean diameter after breakup illustrated in the study of O’Rourke and Amsden [10], and \( q \) value is set to 2.

4. RESULTS AND DISCUSSIONS
With the intention of the comparison between experimental and numerical results of breakup characteristics, such as the deformation rate, the droplet’s trajectory, and droplet size distribution were measured, and the numerical study using the KIVA code was also conducted simultaneously. In the present study, investigation of the breakup characteristics of droplets was performed based on the transition Weber number of breakup regimes listed in Table 2 that is obtained by experiments.

4.1 First breakup stage
After a droplet enters the steady air flow field, the droplet is influenced by the distribution of aerodynamic pressure around the droplet. Since the steady air stream flows around a droplet, the air velocity and aerodynamic pressure distribution at any point on the droplet surface are not uniform. The air velocity vanishes at the droplet pole which becomes the stagnation point. And also the air velocity at the equator has the maximum. According to the Bernoulli’s law, the aerodynamic pressure becomes higher at the pole and lower at the equator, and then the droplet can be deformed from its spherical shape.

Fig. 6 illustrates the experimental and numerical results

| Table 2. Transition Weber number of breakup regimes |
|-----------------------------------|---------------|
| Deformation                      | We < 35       |
| Bag breakup                      | 35 < We < 75  |
| Stretching & thinning breakup    | 75 < We < 350 |
| Catastrophic breakup             | 350 < We      |
of deformation rate according to the Weber number at the first breakup stage. From the results, it can be seen that the flattening occurs rapidly with increase of Weber number. The curve fitted line to the experimental results describes a parabola. This implies that the droplets are flattened due to the external force until the external force is higher than the restoring force caused by viscosity and surface tension, while the restoring force becomes higher when the deformation rate reaches to its maximum.

Additionally, two mathematical breakup models, i.e. TAB and DDB, were used for predicting the deformation rate at the first breakup stage and its results according to the Weber number are shown in Fig. 6. As the results, in the lower range of the Weber number, the TAB model well predicts the deformation rate, but on the other side, the DDB model reasonably matches with the experimental results in the condition of higher Weber number. However, both the TAB and the DDB can’t estimate the maximum deformation rate. Moreover, in the case of the TAB model, the breakup takes place earlier than the practice.

In the first breakup stage, since there is no dispersing process, droplet traveling path that is only influenced by the drag force acting on the droplet can be verified with consideration of its shape and surface area changes. In the present study, two kind of drag model previously discussed, i.e. Eq. (6) of Liu et al. [8] and Eq. (7) of Liu and Reitz [7], are considered for predicting the droplet’s trajectory and its results compared to the experiments are shown in Fig. 7.

According to the information of the Fig. 7, by use of drag coefficient which don’t consider only droplet shape change but increment of droplet surface area, suitable results to the practical trajectory could be obtained with comparison to the result of the drag model in original KIVA code.

4.2 Second breakup stage

As the relative velocity between a droplet and air stream is further increased over the first breakup stage, liquid droplet experiences disintegration. Especially, in this region, droplet encounter the three breakup regimes, so-called bag breakup, stretching & thinning breakup, and catastrophic breakup. In the present study, based on the transition Weber number of breakup regimes, three cases of Weber number, viz. 68, 153, and 383, are considered in both experimental and numerical approaches. Particularly, in the numerical consideration, two mathematical models, drag models, and size distribution models were included as factors for an assessment of predicting accuracy.

Fig. 8 shows the shadowgraphs of the dispersing droplets in a cross flow air stream and its simulation results obtained by using the KIVA code according to Weber number. Before beginning the numerical work, factors for consideration of the size distribution model and drag model were defined as illustrated in Table 3. Just for reference, the TAB model and CASE 1 are originally included in KIVA code. From inspections of Fig. 8, the TAB model seems to well predict the droplet distribution range in the radial direction except a consideration of its size, while the DDB’s droplets are widely distributed. But the DDB model with modified drag model (Liu and Reitz [7]) comes to have suitable broad of droplet existence. In a large Weber number condition, on the other side, all numerical models have some difference in a distribution wide compared to the practice.

By use of the PDPA measurement, droplet size distributions in both radial and axial direction could be obtained. Then, Fig. 9 describes the comparison between the results of droplet size and its calculated data by KIVA code when the Weber number is 68. The experimental results show that droplet size is increased in the radial distance because the large droplet has larger momentum in

<table>
<thead>
<tr>
<th>CASE</th>
<th>Distribution function</th>
<th>Drag model</th>
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<tr>
<td>1</td>
<td>$\chi^2$ distribution</td>
<td>Liu et al. [8]</td>
</tr>
<tr>
<td>2</td>
<td>$\chi^2$ distribution</td>
<td>Liu and Reitz [7]</td>
</tr>
<tr>
<td>3</td>
<td>Rosin-Rammler distribution</td>
<td>Liu and Reitz [7]</td>
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Fig. 8 Experimental and calculated images of breakup behavior of droplets in a cross flow field

Fig. 9 Droplet size distribution in radial direction (Bag breakup)

radial direction than that of the small droplet. In addition, large differences between arithmetic mean diameter (AMD) and Sauter mean diameter (SMD) can be found.

Based on the numerical results, the TAB model well estimates the practical AMD, while the DDB model predicts the larger AMD than that of experiments. In consideration of SMD, besides, the TAB model has large differences with the results of experiment. This implies that the TAB model predicts somewhat small size of droplet and the DDB has a little large droplet. Moreover, droplet size distribution models used in this study can not specify the practical distribution of size.

In a consideration of droplet existence range at $Z = 16$, modified drag model’s results are not reasonable, while the
1. As follows, this work, the conclusions of this study can be summarized.

2. The stream are simultaneously performed. From the results of analyzing the breakup characteristics in a cross flow air stream, it can be understood that the DDB predicts the various size of droplet after breakup.

3. The DDB has large droplet more than the TAB in Fig. 9, it can be also found that CASE 3 makes smaller droplet size. In this conditions, droplet sizes were acquired at the origin of the radial direction, viz. R = 0 because of the short range of droplet existence. From the results, it can be also found that CASE 3 makes smaller droplet size. In CASE 1, the DDB gives the small droplet size in comparison to the other cases. As referenced to the fact that the DDB has large droplet more than the TAB in Fig. 9, it can be understood that the DDB predicts the various size of droplet after breakup.

4. CONCLUSION

In this study, experimental and numerical approaches on analyzing the breakup characteristics in a cross flow air stream are simultaneously performed. From the results of this work, the conclusions of this study can be summarized as follows:

1. At the first breakup stage, the TAB model well predicts the deformation rate in lower range of Weber number, while the DDB model properly matches with the practice in higher Weber number condition.
2. Modified drag model which considers additional variance of the droplet surface area has the suitable results more than the drag model related to the only shape change of a droplet.
3. At the second breakup stage, the TAB included in the original KIVA code, and the DDB with modified drag model are well predict the broad of droplet existence in lower Weber number range. Also, The TAB model predicts somewhat small size of droplet after breakup, and the DDB model makes larger droplets than those of the TAB.
4. Based on the results of size distribution, the DDB model predicts the various size of droplet after breakup than that of the TAB model.
5. Two size distribution models used in this study can not verified the experimental results. Therefore developing a more available distribution model is needed. The reason why the difference of the droplet existence range is closely related to an error of the calculated droplet size to the practice.

5. ACKNOWLEDGEMENT

This study was supported by the CEFV (Center for Environmentally Friendly Vehicle) of Eco-STAR project from MOE (Ministry of Environment, Republic of Korea). Also, this work is financially supported by the Ministry of Education and Human Resources Development (MOE), the...
Ministry of Commerce, Industry and Energy (MOCIE) and the Ministry of Labor (MOLAB) through the fostering project of the Lab of Excellency.

6. REFERENCES